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Topology-preserving simplification of 2D nonmanifold meshes with embedded structures

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Abstract Mesh simplification has received tremendous attention over the years. Most of the previous work in this area deals with a proper choice of error measures to guide the simplification. Preserving the topological characteristics of the mesh and possibly of data attached to the mesh is a more recent topic and the subject of this paper. We introduce a new topology-preserving simplification algorithm for triangular meshes, possibly nonmanifold, with embedded polylines. In this context, embedded means that the edges of the polylines are also edges of the mesh. The paper introduces a robust test to detect if the collapse of an edge in the mesh modifies either the topology of the mesh or the topology of the embedded polylines. This validity test is derived using combinatorial topology results. More precisely, we define a so-called *extended complex* from the input mesh and the embedded polylines. We show that if an edge collapse of the mesh preserves the topology of this extended complex, then it also preserves both the topology of the mesh and the embedded polylines. Our validity test can be used for any 2-complex mesh, including nonmanifold triangular meshes, and can be combined with any previously introduced error measure. Implementation of this validity test is described. We demonstrate the power and versatility of our method with scientific data sets from neuroscience, geology, and CAD/CAM models from mechanical engineering.

Keywords Computational geometry and its applications · LOD techniques · Multiresolution curves and surfaces

1 Introduction

We assume the reader is familiar with mesh simplification techniques in general. Most of the previous works on mesh simplification are devoted to the development of a specific scalar error measure, taking into account the geometry of the mesh and possibly the values of data attached to the mesh. A more recent topic in mesh simplification is the preservation of topological characteristics of the mesh and of data attached to the mesh. In this paper a new topology-preserving simplification algorithm is introduced for triangular meshes, possibly nonmanifold, in which polylines are embedded. In this context embedded means that the edges of the polylines are also edges of the mesh. The mesh is simplified by repeated edge collapses, following the classical scheme introduced in previous works. For each edge of the mesh, the cost of its collapse is computed and inserted in an ordered heap of edges. The algorithm iteratively pops the edge collapse, introducing the lowest error in the simplified mesh, checks for the validity of the collapse (i.e., geometry, topology, or attribute consistency), and, if this operation is accepted, updates both the mesh and the heap. Since the polylines form a subset of the edges of the mesh, edge collapses modify both the mesh and the set of polylines embedded in the mesh. The main contribution of this paper is a robust validity algorithm that detects whether or not an edge collapse preserves the topology of the mesh *as well as* the topology of the embedded polylines. To this end we define a so-called *extended complex* that implicitly encodes both topologies. Thereafter edge collapses that preserve the topology of this extended complex are detected. We show that these edge collapses preserve also both the topologies of the mesh and the embedded polylines. As we will see in this introduction, the preservation of these topological characteristics is crucial in several applications.

Many application areas may produce meshes with embedded polylines. This work has been motivated by three applications, for which the topology of the mesh and the topology of the embedded polylines must be preserved throughout the simplification.

Neuroscience

One of the great challenges of these years, in neuroscience, is to automate the *brain mapping*. As annotating brains is complicated and time consuming, neuroscientists are working on a unified atlas created from a collection of different brains that will later be used to directly transfer information to the patient. One method of mapping brains is to segment the cortical surface according to the main features (i.e., the gyri and the sulci) and to match these features. This segmentation produces polylines surrounding these features on the cortical surface. The topology of the polylines is simple: they form closed curves without self-intersections. Due to the high complexity of the surface, preprocessing needs to be done. Simplifiying the surface while preserving the main folds is one possible method. In this context it is critical to preserve the topology of the polylines and the mesh: the mesh should remain a manifold surface, and the polylines should remain closed curves without self-intersections.

Geology

Understanding soil quality is fundamental for agriculture or estimating the health of the environment. Soil surveys are presented as maps encoding the proportion of each soil within a specific area. The boundaries of these domains are really important because they will be used to drive a specific land use (e.g., oil exploitation). In terms of visualization, data reduction is often used because of the difference between the area of interest (e.g., country) and the size of one soil type. While these data are characterized by a fairly simple surface representing a height field, embedded structures such as the boundaries among soil types can be very complex. It is crucial to preserve the topology formed by these boundaries in order to properly recover the different soil types in the simplified model. Also, in a volume these layers of soils create multiple surfaces intersecting each other, which can be seen as a complex nonmanifold surface.

CAD/CAM

CAD/CAM models are based on a geometry produced by a modeler and adapted to the design of a mechanical model. Handmade geometry can be highly complex, and features such as materials can be added to the mesh. In addition, there is a coherence between the material interfaces and the geometric features that is important for the consistency of the whole CAD/CAM model. 2D numerical simulations based on FEMs (finite element methods) on a CAD/CAM model produce a triangular mesh, possibly nonmanifold, in which the edges follow the geometric features of the CAD/CAM model as well as the interfaces separating the different materials. Preserving the topology of the polylines formed by these characteristic edges is essential to maintain the consistency of the CAD/CAM model throughout the simplification.

This paper is structured as follows. Section 2 reviews related works and points out differences with the present paper. Section 3 reviews the previously introduced topological tests for edge collapses in 2-complexes, as these results are used to detect valid edges in the extended complex. Section 4 explains how the extended complex is defined and describes the actual implementation of the validity algorithm. Finally, Sect. 5 presents results in the different application areas cited above, and Sect. 6 gives a conclusion with directions for future work.

2 Related work

Research on surface simplification in computer graphics and scientific visualization has led to a substantial number of methods within the last 12 years. An exhaustive description of this field is beyond the scope of this paper, and one can refer to the many surveys [2, 10, 24] available. In the remainder of this section we only review the methods relevant to our work.

2.1 Surface simplification

In order to decimate a discrete surface, different operators can be applied on its elements (i.e., vertex, edges, faces). Historically, the first method was region merging [17], followed by numerous methods such as surface retiling [31], vertex decimation [30], vertex clustering [28], subdivision meshes [7], and wavelet decomposition [15]. Among these decimation methods the local iterative edge collapse operator has been widely used in order to control precisely the simplification error. These methods depend largely on the metric used to determine the order of the modifications, and many heuristics have been proposed depending on the application. Particularly relevant algorithms used progressive meshes [18, 26], tolerance volumes [16], plane deviations [27], quadric-based metrics [11], or memoryless simplification [22]. While this wide range of methods deals well with geometry, models used in computer graphics or in solid modeling may have different nongeometric attributes defined on the surface that need to be preserved through the simplification process.

2.2 Attribute-preserving simplification

Discontinuities are often observed in the nongeometric attribute field of a surface such as high color variation due to shadows projected on a surface or texture coordinates defined on vertices. Attribute errors can be estimated on each face to compute an error-bounded simplification [1], treated with dedicated datastructures such as *wedges* [19] or integrated into the computation of a quadric error metric [12]. Erikson and Manocha [8] track attribute variations by minimizing an error computed on point clouds defined by these field values. A resampling of the attributes can also be used as in [3], where a displacement map is computed on high resolution and applied as a texture on the simplifed mesh. Cohen et al. [4] dynamically compute a *texture deviation metric* used to decouple the sampling of the attributes and to ensure an error bound of the simplification. In some cases, attributes can describe geometric features of a surface, but usually specialized structures and algorithms are required, especially when these features are defined in different dimensions.

2.3 Application-driven simplification

Feature-guided simplification provides dedicated error metrics to integrate strong constraints on subsets of the original model such as the intersection of roads on a terrain data. These metrics depend largely on the application because the definition itself of a feature is based on the field of study. Many methods have been proposed to extract a collection of piecewise linear curves on a surface [21, 25, 29]. Once such features have been extracted, associated weights can be used to integrate them in any kind of error measure previously presented like that in [32], which extends the quadric-based error metric of [11]. User-guided simplification also provides specific error metrics to simplify chosen features.

Terrain simplification is a standalone branch of mesh simplification as it has been studied for almost 30 years and has led to an incredible number of methods. Among others, [9, 23] provide a survey of this wide field. Several papers deal with the preservation of the topology of polylines on these terrains. However, due to the nature of terrain data, they are restricted to height field meshes having a very simple topology. All these methods are either extensions of the Douglas–Peucker [6] algorithm (which does not deal with topology) for curve simplification or are based on heuristics defined by the local topology around the polyline vertices.

Material boundaries are special properties of CAD/CAM models. Preservation of the material interface through simplification in CAD/CAM applications has been mentioned in some previous works, including [14] for volume data. In these cases, the interface is integrated in a scalar attribute such as color, and the global geometric appearance is maintained through simplification. Preservation of the topology of a scalar field and isosurfaces defined on this field has been considered and guaranteed in several methods such as that in [13].

However, in our framework we are interested in preserving the topology of such properties defined for the model (i.e., patches of surfaces), for the material boundaries (i.e., subsets of the mesh edges), and for the intersections among boundaries (i.e., specific vertices of these boundaries). Existing methods lack a unique combinatorial validity test preserving the topology of all the elements defined in all dimensions lower than or equal to 3.

2.4 Topology-preserving simplification

Surface mesh simplification algorithms transform a general simplicial complex into another one. Edge collapse is the most widely used operator for removing subsimplicies from input because it allows one to control efficiently and accurately the deformation introduced in the initial mesh. Some of these algorithms preserve the topology of the simplified mesh by investigating the organization of the triangles around an edge leading to a certain number of cases. In some situations the edge collapse is rejected as it would introduce topological errors [20]. A similar treatement can be performed on feature lines based on local tests in between the edges around a collapse and the triangulation. However, nonmanifold surfaces cannot be treated with this method and there is no obvious extension to higher dimensions, in contrast with our results, as pointed out in Sect. 6.

More recently, Dey et al. [5] have proven that the complex obtained after an edge collapse is homeomorphic to the first one if the neighborhood of the edge collapse satisfies the *link condition*. This result, based on concepts of computational topology, has found increasingly widespread use, even in volume simplification, as it is not restrictived to 2-complexes. Especially, nonmanifold surfaces can be treated with this method using the link condition.

3 Preserving the topology of 2-complexes

Appendix A reviews the basic definitions from combinatorial topology needed in the following sections. Given a simplicial complex, and an edge (a 2-simplex) in this complex, detecting if the collapse of this edge modifies the topology of the complex is a challenging task and is extensively studied in [5]. For 2- and 3-complexes, necessary and sufficient conditions have been developed ensuring that a continuous one-to-one mapping between the complex before edge collapse and the complex after edge collapse exists. These conditions are slightly more restrictive than topology preservation since the mapping is required to be the identity outside a neighborhood of the edge being contracted. In this section we state the results developed in [5] for 2-complexes, as they will be used later in this paper. The reader is referred to the original paper for details.

3.1 Order of a simplex in a 2-complex

The order of a simplex τ in a complex K measures the topological complexity of the neighborhood of τ in K. It is denoted by ord τ. For a 2-complex, the order of simplices may be 0, 1, or 2. A simplex τ has an order 0 if St τ is homeomorphic to an open disc (see Appendix A for definitions). This is the simplest case, where locally the complex is a 2-manifold. The order is 1 if St τ is homeomorphic to p triangles sharing a common edge with $p \neq 2$. This is, for example, the case for nonmanifold edges and boundary edges. In all other cases the order of the simplex is 2. Note that all triangles have order 0 and that edges may have order 0 or 1 only. Figure 1a illustrates the order of vertices and edges in a 2-complex.

Boundaries of a 2-complex. Topological tests rely on a generalized notion of a boundary. The jth boundary in a 2-complex K is defined as the set of all simplices with order j or higher:

$\mathrm{Bd}_j\ K = \{\tau \in K \mid \mathrm{ord}\ \tau \geq j\}$.

The 0th boundary of K is K. Figure 1b illustrates the first and second boundaries of a 2-complex. Note that the

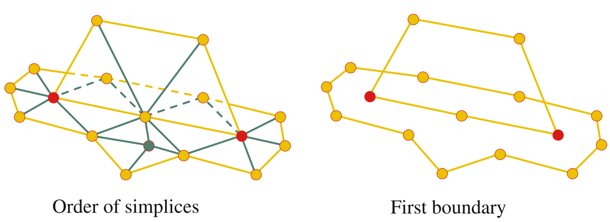

Fig. 1. a Order of simplices in a 2-complex: the *green simplices* have order 0, the *yellow simplices* have order 1, and the *red vertices* have order 2. **b** Edges and vertices in first boundary of 2-complex shown in **a**. The two *red vertices* form the second boundary of this complex

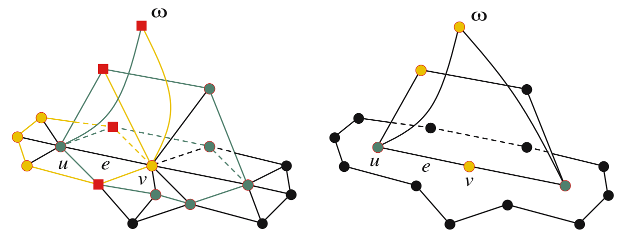

Fig. 2. Topological test for an edge $e = uv$: the *left* and *right* parts illustrate, respectively, the first and second conditions. *Left*: $\mathrm{Lk}_0^\omega\ u$ contains the *yellow* and *red vertices* and the *yellow edges*, $\mathrm{Lk}_0^\omega\ v$ contains the *green* and *red vertices* and *green edges*, $\mathrm{Lk}_0^\omega\ e$ contains the *red vertices*: the first condition is fulfilled. *Right*: $\mathrm{Lk}_1^\omega\ u$ contains the three *yellow vertices*, $\mathrm{Lk}_1^\omega\ v$ contains the two *green vertices*, the intersection is empty: the second condition is fulfilled

first boundary contains not only the usual boundary edges but also nonmanifold edges adjacent to three triangles or more. Since only vertices may have order 2, the second boundary of a 2-complex is always a set of vertices.

Topological test. Let ω be a dummy vertex, define $K^\omega = K \cup (\omega \cdot \mathrm{Bd}_1\ K)$ and $G^\omega = \mathrm{Bd}_1\ K \cup (\omega \cdot \mathrm{Bd}_2\ K)$. See Appendix A for the definition of the cone $\omega \cdot T$, where T is a set of simplices. Let $\mathrm{Lk}_0^\omega\ \tau$ and $\mathrm{Lk}_1^\omega\ \tau$ denote the link of τ in K^ω and G^ω, respectively. With all these definitions at hand, one can now state the main result of [5] for 2-complexes.

Topological test:

Theorem 1. *Let K be a complex and L be the complex obtained by contracting the edge uv in K. The following two conditions ensure that L has the same topology as K:*

(i) $Lk_0^\omega\ u \cap Lk_0^\omega\ v = Lk_0^\omega\ uv$,
(ii) $Lk_1^\omega\ u \cap Lk_1^\omega\ v = \emptyset$.

Conditions (i) and (ii) are illustrated in Fig. 2 for a nonmanifold edge.

4 Topology-preserving edge collapse with embedded structures

Let K be a 2-complex and E a set of edges in K. The closure $\overline{E}$ of E is a collection of edges and vertices that may be viewed as a set of several polylines embedded in K, with possible self-intersections. These polylines together with the polylines formed by all boundary and nonmanifold edges of K define a 1-complex $F = \overline{E} \cup Bd_1 K$ whose topology is an important characteristic of the data and should be preserved. Contracting edges in K possibly modifies both the topology of K and the topology of F. Our goal is to develop a *robust* test to detect which edge collapse preserves the topology of the 2-complex K *together* with the topology of the polylines embedded in

K. The key idea of the paper is to implicitly encode the topology of the polylines F and of the embedding complex K in a single extended 2-complex $\widetilde{K}$. Thereafter the topological test developed in [5] (Sect. 3) can be used to select the valid edge collapses in the extended 2-complex $\widetilde{K}$. $\widetilde{K}$ is built in such a way that valid edge collapses in $\widetilde{K}$ preserve the topology of both K and the embedded polylines F.

4.1 Implicitly encoding the topology of the embedded structures

We can assume that the collection of edges E contains only order 0 edges, without changing F: if there is an order 1 edge in E, it is also in $Bd_1 K$. The extended complex $\widetilde{K}$ is built by adding to K the cones from the dummy vertex ω to each edge in E. More precisely:

$$\widetilde{K} = K \cup w \cdot \overline{E}\,.$$

In other words, the embedded edges are extended to a 2D subcomplex of the extended complex $\widetilde{K}$. Figure 3 illustrates an extended complex with two intersecting embedded polylines. The extended complex $\widetilde{K}$ is defined such that its first boundary equals F:

Lemma 1. $Bd_1 \widetilde{K} = F$.

Proof. An edge in E has an order 0, i.e., it has exactly two adjacent triangles in K and three adjacent triangles in $\widetilde{K}$. Thus it has an order 1 in $\widetilde{K}$. An edge of order 1 in K is not in E; therefore, it has the same star in K and $\widetilde{K}$ and is still of order 1 in $\widetilde{K}$. Thus we have $F \subset Bd_1 \widetilde{K}$. Now let e be an edge of order 1 in $\widetilde{K}$. e has one or three or more triangles in its star. If e has only one triangle in its star, it is in the boundary of K. If it has three or more triangles in its star, it is either a nonmanifold edge of K or an edge in E. Thus we have also $Bd_1 \widetilde{K} \subset F$. □

Lemma 1 means that the topology of the embedded structure F is encoded implicitly in the extended complex $\widetilde{K}$. An edge collapse in K corresponds to an edge collapse in $\widetilde{K}$. After an edge collapse, the modified set of polylines F can be retrieved as the set of edges facing the dummy vertex ω in a triangle. Lemma 2 below, applied to the extended complex $\widetilde{K}$, proves that if an edge collapse preserves the topology of $\widetilde{K}$, it also preserves the topology $Bd_1 \widetilde{K}$, i.e., it also preserves the topology of the modified set of polylines F, since $Bd_1 \widetilde{K} = F$ from Lemma 1.

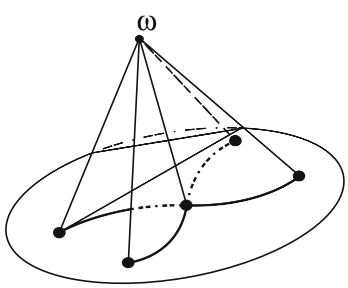

Fig. 3. Extended complex: the extended complex is defined by adding to the mesh all edges and faces connecting a dummy vertex ω with the vertices and edges of the embedded polylines

Lemma 2. *Let K be a 2-complex and uv an edge in K such that (i) and (ii) are satisfied. Then the collapse of uv preserves the topology of $Bd_1 K$.*

Proof. Let

$$M = Bd_1 K, \text{ and } M^\omega = Bd_0(M) \cup \omega \cdot Bd_1(M)\,. \qquad (1)$$

M is a 1-complex: it is the closure of the set of order 1 edges in K. Since M is a 1-complex, it is sufficient to show that there are no vertices in the intersection of the links of u and v in M^ω. Since $Bd_1 M = Bd_1(Bd_1 K) \subset Bd_2 K$, any vertex in the link of u (resp. v) in M^ω is also in the link of u (resp. v) in G^ω. Therefore, condition (ii) implies the lemma. □

4.2 Implementation of the topological preservation test

In this section we describe the actual implementation of the algorithm that detects if an edge $e = uv$ can be collapsed without changing the topology of the surface or the topology of any polyline defined on the triangle edges.

1. **Initialization:** Set up local variables in the neighborhood of the two vertices u and v of edge e. For each edge e_i adjacent to u or v, add dummy triangles ($\omega \cdot e_i$) if e_i is either on a polyline or in the first boundary of the mesh. This amounts to locally computing the extended complex.
2. **Compute the vertices in $\mathbf{Lk}_0^\omega\, u$:** Loop through edges e_i adjacent to u in the extended complex. For each edge e_i, store the vertex facing u in $\mathrm{Lk}_0^\omega\, u$. In some cases the opposite vertex is the dummy vertex ω if the edge is in a polyline or in the first boundary of the mesh. This case is implicit and is not treated explicitly as the neighborhoods of u and v have been triangulated by adding dummy cells using the dummy vertex ω.
3. **Compute the edges in $\mathbf{Lk}_0^\omega\, u$:** Loop through the faces f_i adjacent to u in the extended complex. For each face f_i, store the edge facing u in $\mathrm{Lk}_0^\omega\, u$. Figure 4 illustrates the computation of the link around a nonmanifold vertex.
4. **Compute $\mathbf{Lk}_0^\omega\, v$:** The exact same process is applied around v in order to compute $\mathrm{Lk}_0^\omega\, v$.
5. **Compute $\mathbf{Lk}_0^\omega\, e$:** Loop through the faces f_i adjacent to e in the extended complex. For each face f_i, store the vertex facing e in $\mathrm{Lk}_0^\omega\, e$.
6. **Compare $\mathbf{Lk}_0^\omega\, u \cap \mathbf{Lk}_0^\omega\, v$ and $\mathbf{Lk}_0^\omega\, e$:** The intersection of $\mathrm{Lk}_0^\omega\, u$ and $\mathrm{Lk}_0^\omega\, v$ is computed. If this intersection contains an edge, the edge collapse is rejected, as $\mathrm{Lk}_0^\omega\, e$

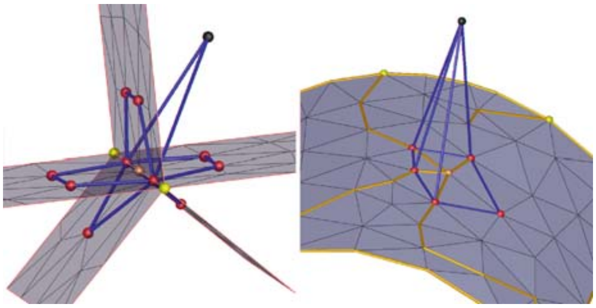

Fig. 4. Link of a nonmanifold vertex on the intersection of several surfaces in the extended complex (*left*). Link of a vertex on the intersection of several polylines. In the extended complex, dummy triangles are added from the segment polylines modifying the link of the vertex as shown (*right*). *Yellow vertices* are intersections of *orange polylines*. *Red vertices, blue edges*, and the *black dummy vertex* are simplices of the link of the *orange vertex*

contains only vertices. Compare the number of vertices in $\mathrm{Lk}_0^{\omega}\, u \cap \mathrm{Lk}_0^{\omega}\, v$ with the number of vertices in $\mathrm{Lk}_0^{\omega}\, e$. The edge collapse is rejected if these numbers are not equal. It is sufficient to compare the number of vertices, and not the actual indices of the vertices, since in any case the vertices in $\mathrm{Lk}_0^{\omega}\, e$ form a subset of the vertices in $\mathrm{Lk}_0^{\omega}\, u \cap \mathrm{Lk}_0^{\omega}\, v$.

7. **Compute $\mathbf{Lk}_1^{\omega}\, u$:** Loop through edges e_i adjacent to u and in the first boundary of the extended complex. For each edge e_i store the vertex facing u in $\mathrm{Lk}_1^{\omega}\, u$. Note that if u is in the second boundary of the mesh, then edge ωu is in G^{ω} (more precisely in $(\omega \cdot \mathrm{Bd}_2\, K)$), and thus the dummy vertex ω is in $\mathrm{Lk}_1^{\omega}\, u$.
8. **Compute $\mathbf{Lk}_1^{\omega}\, v$:** The exact same process is applied around v computing the $\mathrm{Lk}_1^{\omega}\, v$.
9. **Compute $\mathbf{Lk}_1^{\omega}\, u \cap \mathbf{Lk}_1^{\omega}\, v$:** The intersection of $\mathrm{Lk}_1^{\omega}\, u$ and $\mathrm{Lk}_1^{\omega}\, v$ is computed. If this intersection is not empty, then edge e is rejected for collapse as the second link condition breaks.
10. **Accept collapse of e:** If edge e has not been rejected in previous steps, it can now be contracted. The dummy cells introduced in the first step are removed.

5 Applications and results

We have applied our topology-preserving simplification algorithm on a wide range of datasets. The error measure used in all examples to sort the edges in the heap is a simple weighted combination taking into account the maximal normal deviation of the faces and the polyline's edges, before and after collapse. This simple error measure already gives satisfactory results, but we stress the fact that our algorithm can be combined with any error measure. All simplifications run at about 20,000 removed edges per second on a 1.7-GHz Pentium M laptop. In the following section, all decimation percentages are taken from the original number of triangles in the mesh. The first paragraph shows a neuroscience application involving a complicated surface geometry but with a very simple feature line topology. Then a CAD/CAM application is presented illustrating both a complicated geometry and feature line topology. Finally, geological data allow us to validate our method with a simple geometry but an extremely complex polyline topology.

Geology data with nonmanifold models. As our edge-collapse criteria for preserving topology is valid for 2-complexes, nonmanifold models can directly be simplified with the same topological test. Figure 5 shows the simplification of geological data. Several layers separating different soil types create a nonmanifold surface. Linear features are present in the model that give specific information on a subset of the data and thus need to be handled in the simplification process. Even with a high simplification ratio of 95%, the topology of the nonmanifold surface is preserved as well as the linear features embedded on the triangle edges. Note that several polylines are crossing a nonmanifold region creating vertices of order two that are successfully preserved through the collapses.

Neuroscience. We have applied our simplification algorithm on a triangle mesh extracted from MRI data and segmented according to the sulci and the gyri (Fig. 6a,b,e,f). Thus we were able to generate a low-resolution mesh while preserving its main features: the mesh remains a manifold surface, and the polylines remain closed curves without self-intersections. The closeup view shows the effect of the simplification on a feature. Note that the boundaries are preserved despite the reduction of vertices along them. As many features can be small and close to each other, allowing the collapse of edges around and along the boundaries is essential to keep the simplification rate low.

CAD/CAM model with complex polylines. Figures 6c,d,g,h and 7 show the results on a CAD/CAM model of a piston after a simplification ratio up to 98%. The solid views show the high simplification ratio applied to the triangles, while the polyline views exhibit the preservation of both the geometry and the topology of the polylines. In this particular example polylines are extracted as interfaces in between materials defined on the surface. Even with a high simplification ratio of 98%, the interfaces are not altered, as illustrated in the closeup views.

Terrain data. Figure 8 shows a subset of the surficial materials of Canada. Boundaries among soil types are represented by red polylines. By applying our algorithm we successfully simplify the surface down to one fifth of the original data without affecting the topology of the polylines. The simplification process is stopped when no edges

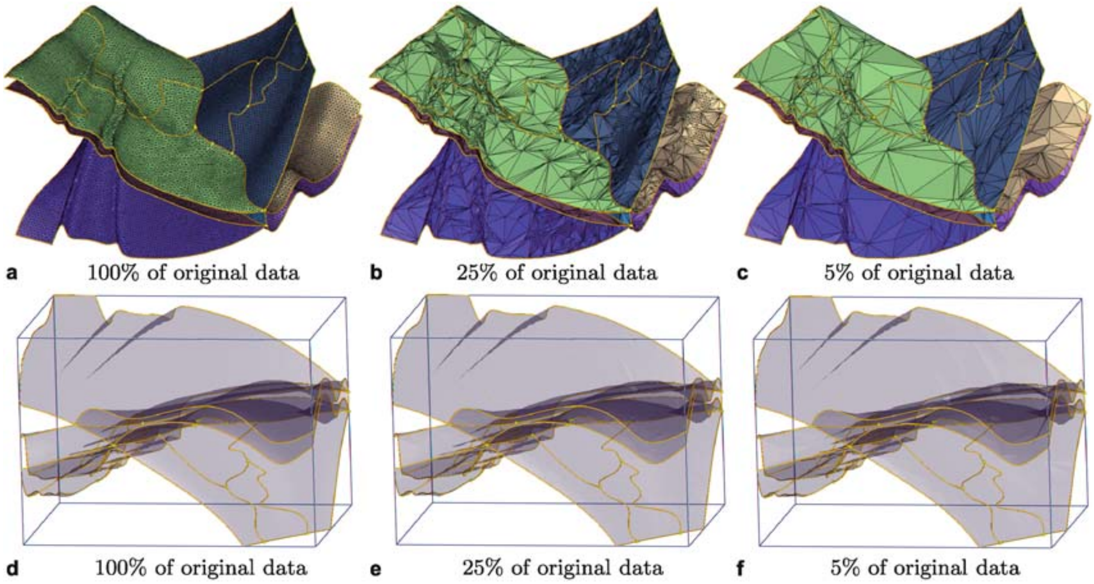

Fig. 5. Geological data showing several surfaces separating different soil types. Linear features are present in the data and on the layers. This set of interfaces among soil types creates nonmanifold surfaces

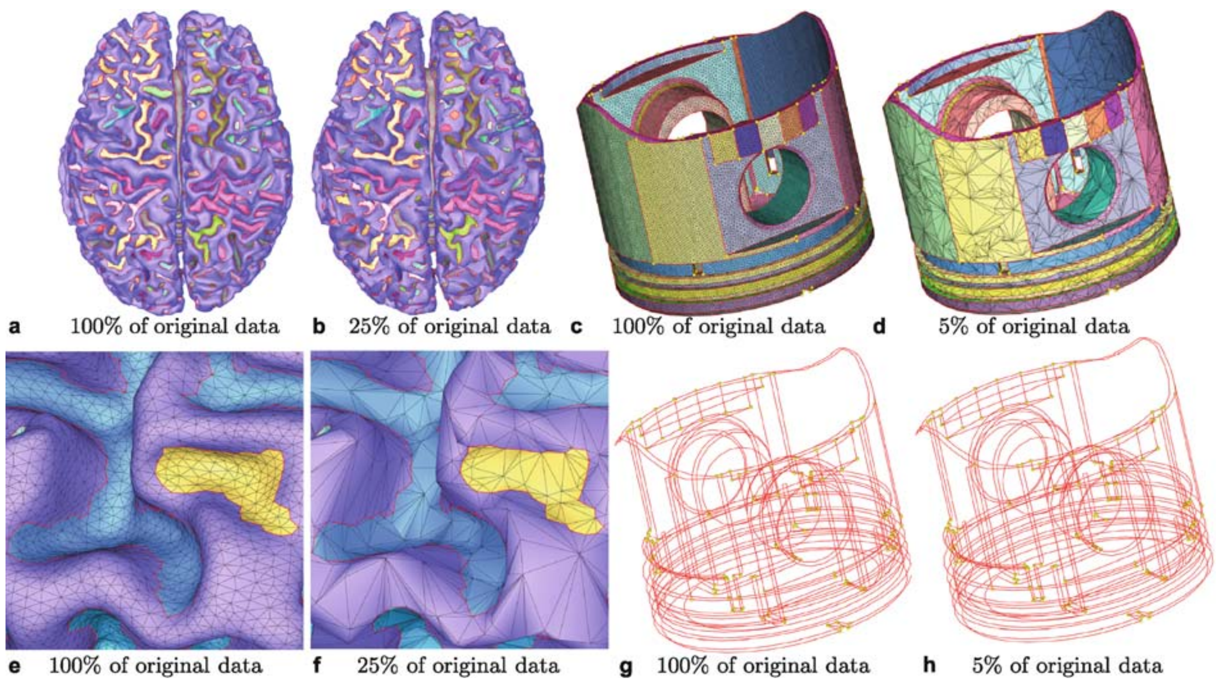

Fig. 6. (**a**, **b**, **e**, **f**) Segmented surface of a human cortex. The sulci and gyri of the cortex (i.e., main features) are preserved by the simplification. (**c**, **d**, **g**, **h**) CAD/CAM model of a piston simplified by preserving its material properties. *Top row*: solid faces of model with material boundaries defined as polylines. We ensure that the topology of these polylines remains unchanged. *Bottow row*: the main features of the model are preserved even after removing 95% of the vertices

can be removed without introducing a certain deviation into the polylines. The simplification process stops early because of the complexity of the polylignes and not because of changes in the surface topology. The closeup view shows the simplification emphasizing the complexity of the soil structure. Note that intersections among polylines are represented by yellow spheres and that no such features are removed through the simplification (i.e., it

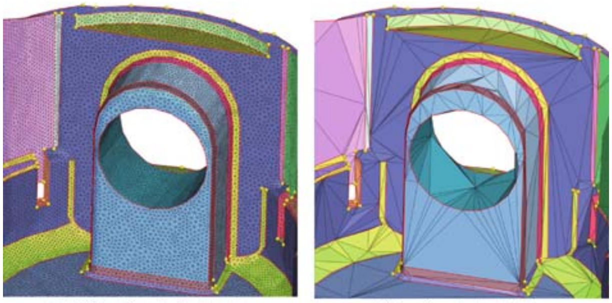

Fig. 7. CAD/CAM model of a piston simplified by preserving its material properties

would introduce a topological modification of the polylines).

6 Conclusion

This paper has introduced a new topology-preserving simplification algorithm, based on edge collapse, for nonmanifold triangular meshes with embedded polylines. The topology of the polylines as well as the mesh is preserved throughout the simplification using a *unified and robust validity algorithm* to select edges that can be collapsed. Our algorithm is based on powerful combinatorial topology results and can be applied to general triangular meshes, including nonmanifold meshes. The idea of the *extended complex*, encoding implicitly the topologies of the mesh and the polylines in a unified way, is at the heart of our results. One of the most appealing feature of this idea is its possible extension to 3-complexes. We are currently working on this in order to generalize this concept to tetrahedral meshes with embedded 2D and 1D structures.

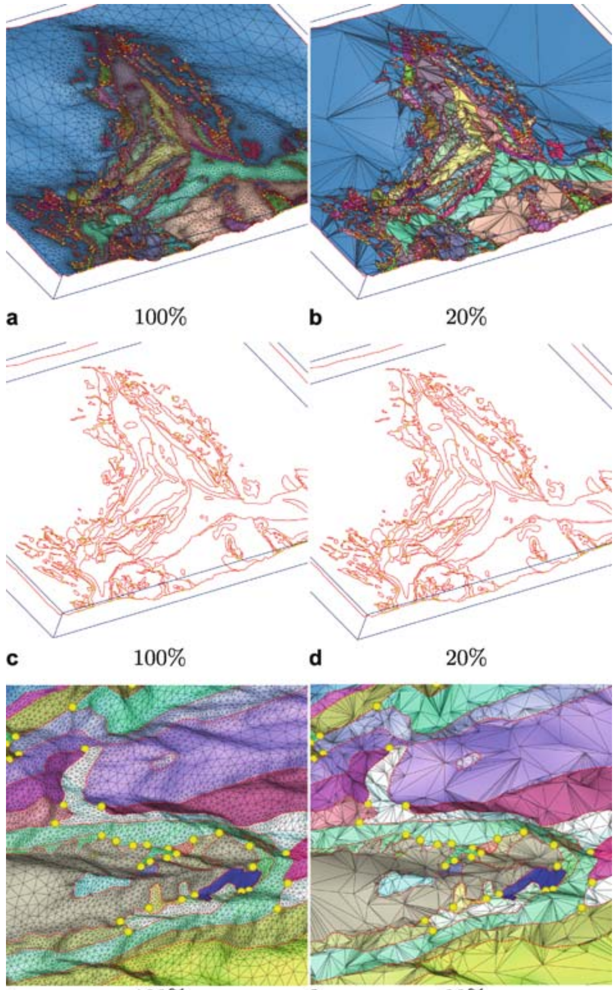

Fig. 8. Subset of surficial materials of Canada. The percentages are ratios of original data. Each soil type is color-coded. Interfaces among them are shown by *red polylines*. Note the complexity of the polyline topology emphasized by *yellow spheres* showing all polyline intersections

Appendix A

A $n-$simplex τ is the set of convex combinations of $n+1$ affinely independent points called the vertices of τ. A simplex η whose vertices are a subset of the vertices of τ is called a face of τ. This is denoted $\eta \leq \tau$ or $\tau \geq \eta$. If η is a face of τ, then τ is called a coface of η. A complex K is a collection of simplices such that:

(i) If $\tau \in K$, then all faces of τ are also in K.
(ii) If $\eta, \tau \in K$, then $\eta \cap \tau$ is empty or a face of η and τ.

The star, the closure, and the link of a set of simplices T in a complex K are denoted, respectively, by St (T), $\overline{T}$, Lk (T) and defined by:

$$\mathrm{St}(T) = \{\eta \in K | \eta \geq \tau \in T\},$$

$$\overline{T} = \{\eta \in K | \eta \leq \tau \in T\},$$

$$\mathrm{Lk}(T) = \overline{\mathrm{St}T} - \mathrm{St}\overline{T}.$$

If τ is a simplex and ω is a point affinely independent of the vertices of $v_1, \cdots, v_n$ of τ, then the cone from ω to τ is denoted by $\omega \cdot \tau$ and defined as a simplex with vertices $\omega, v_1, \cdots, v_n$. If T is a set of simplices, then the cone from ω to T is the union of the cones from ω to τ, with $\tau \in T$. It is denoted by $\omega \cdot T$.

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