Topological Data Analysis of 3D Ablative Rayleigh-Taylor Instability Dataset for Automatic Segmentation

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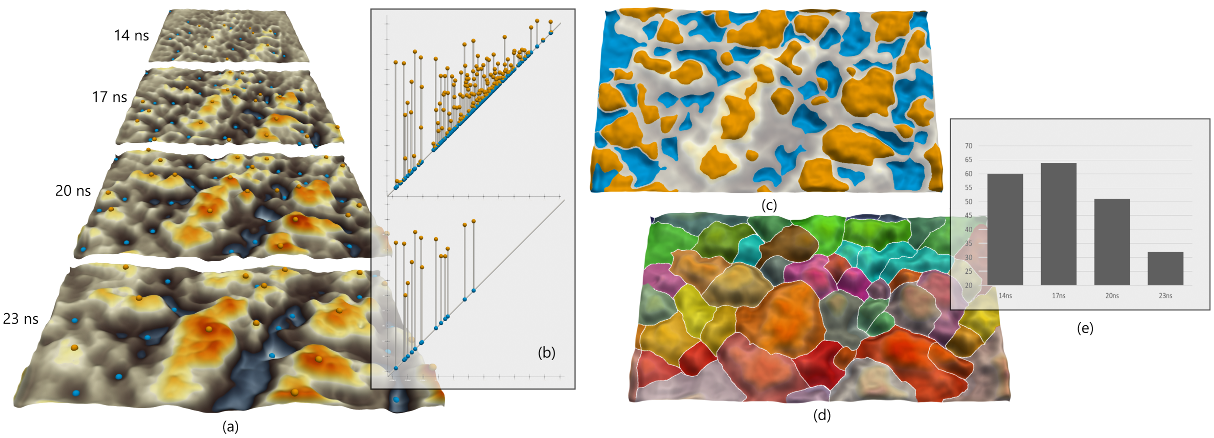

Figure 1: (a) Terrain views of the N180213-001 x-ray dataset, at 4 time steps, obtained experimentally on the National Ignition Facility showing the most critical points corresponding to the the x-ray intensity maxima. (b) Persistence diagrams computed on the original image intensity above and simplified diagram based on a persistence threshold used to extract the critical point of the terrain views. (c) Segmentation of extrema area, due to bubbles related to the hydrodynamic instabilities, generated from the leaves of a contour tree after a topological simplification based on the persistence. (d) Topological segmentation of the bubble region based on the separatrices of an ascending Morse-Smale complex simplified according to a persistence threshold. (e) Distribution of the number of features segmented by the Morse-Smale complex over time, confirming the decreasing number of bubbles into the hydrodynamic instability.

Abstract

This ongoing project aims to provide topological analysis tools to explore hydrodynamics instabilities observed through x-ray acquired during experimental campaigns on the National Ignition Facility (NIF). Topological Data Analysis (TDA) techniques will be evaluated to aid the segmentation of bubbles produced by fluid mixing during the development of Rayleigh-Taylor instabilities. The focus of this work is to evaluate topological tools and provide scientists from different laboratories with ready-to-use visualization topological pipelines to better understand experimental hydrodynamic instabilities.

Index Terms: Human-centered computing—Visualization—Visualization application domains—Scientific visualization

1 Hydrodynamics instabilities on the NIF

Hydrodynamic instabilities [4] that arise during the convergence and implosion phases of Inertial Confinement Fusion (ICF) capsules are one of the major drawbacks [14] on the path to the higher energy gains [1] needed for future inertial fusion energy power plants. Dedicated Discovery Science campaigns [6] have therefore been conducted at the world' largest laser facility, the National Ignition Facility [18] in order to better understand the growth [5] and possibly control mechanisms of these detrimental instabilities. In particular, when starting from imprinted perturbations imposed by the laser focal spots [7] the ablative Rayleigh Taylor instability gives birth at its highly nonlinear stage to complex 3D patterns. Typical data analysis methods based on threshold and watershed segmentation [16] have been used to detect the peak-to-valley differences and the area of the bubbles. Such methods rely heavily on parameters defined by the observers. Therefore, we explore more robust methods based on Topological Data Analysis.

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2 Topological Data Analysis

Topological Data Analysis [8, 19] is a field at the interface of mathematics and computer science that provides robust and efficient techniques for analysing complex phenomena in scientific data. TDA helps to identify hidden patterns in the structure of the data and to facilitate the comparison between members of ensembles. In particular, Persistent Homology [9] measures topological features of shapes and of functions by tracking topological changes of a changing space. The utility of TDA has been already been demonstrated in many fields [12], with examples of successful applications in medical imaging [17], fluid dynamics [15] or combustion [11].

The pixel representation of the x-ray experimentation on the

NIF can be represented as a scalar field. There are many tools in the literature related to topological methods for scalar data. One popular topological representation is the persistence diagram [8] which shows the birth and death of certain features of interest along a filtration. Merge and contour trees [2] describe the connectivity evolution of level sets. The Morse-Smale complex [10] is another structure that uses integral lines to describe area of a manifold in term of its monotonic behaviour. Persistence diagrams are widely used to describe features and one needs metric to compare them. Thus many distances between the diagrams have been studied such as the bottleneck distance or the Wasserstein distance [8].

3 Contributions

We explore several dataset such as the NI180212 or NI180213 through different visualization pipelines in the Paraview software [3]. The topological analysis was done thanks to the Topology Toolkit [20]. First we iterate on the x-ray images to better understand the correlation between the pixel intensity and the bubble structure created during the development of the Rayleigh-Taylor instability. We then construct a representation of the images as a discrete scalar field. The structured pixel image is triangulated to create a triangular mesh defining a manifold surface. The cell centered pixel values are interpolated on the mesh vertices. The size of experimental images of this study is 900×500. From that we use algorithms from the TDA to extract some critical points of interest as shown Fig. 1 (a) by keeping all extrema pairs above a persistence threshold of 0.2 thanks to the persistence diagram shown on Fig. 1 (b) (all pairs near the diagonal have been removed).

Several topological representations have been evaluated in order to segment the features of the x-ray images corresponding to the bubble. First the discrete scalar field representing the pixel values is simplified according to a persistence threshold to preserve the main structures and remove small variations around bubbles. Then a contour tree is built on the scalar field and only the leaves of the trees are kept to segment the valleys (blue regions of Fig. 1 c) and the peaks (orange regions of Fig. 1 c) of the corresponding terrain. We also evaluate a segmentation based on a Morse-Smale complex as shown in Fig. 1 d. We segment the x-ray by the 1-separatrices of the ascending Morse-Smale complex which leads to a very promising segmentation. To go further we compare the number of features segmented at each iteration and compare them with studies such as [13]. These topological structures successfully capture the expected decreasing number of bubbles of the instability over time as shown on the histogram Fig. 1 (e).

4 Conclusion and Future Work

We demonstrate that TDA of 3D Rayleigh-Taylor experimental radiographs is a promising approach to automatically analyze complex patterns in ICF. We show that Contour trees and Morse-Smale complexes extracted from a simplified x-ray based on persistence give promising segmentations. We are now working on bubble association over time with algorithms based on the distances between merge trees. We are also investigating other distances between segmentations such as the the Wasserstein distance between persistence diagrams. These feature tracking strategies will be extended to other experimental radiographs, as well as complex 3D numerical simulations.

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